EXAMPLE 1: A & B are made by sequential dilution of Stock (1.50±0.01 M) – 5.00±0.03 ml of previous solution made up to 25.0±0.2 ml.

c = Moles added (n = c*V) / Volume made up to = $c_{PreviousSoln}V_{PreviousSoln}/V_{Tot}$

$c_A = C_{Stock}V_{StockUsed}/V_{Tot} = 1.5*5/25 = 0.3$M $c_B = C_AV_{AUsed}/V_{Tot} = 0.3*5/25 = 0.06$M

$c_{Max} = C_{PrevSoln,Max}V_{PrevSoln,Max}/V_{Tot,Min}$ $c_{Min} = C_{PrevSoln,Min}V_{PrevSoln,Min}/V_{Tot,Max}$

$c_{A,Max} = 5.03*1.51/24.8 = 0.3063$M $c_{A,Min} = 4.97*1.49/25.2 = 0.2939$M

$c_{B,Max} = 5.03*0.3063/24.8 = 0.0621$M $c_{B,Min} = 4.97*0.2939/25.2 = 0.0580$M

Δ_A: (0.3063-0.3000 =) 0.0063 & 0.0061 Δ_B: 0.0021 & 0.0020

$c_A = 0.3000 \pm 0.0063$ M $c_B = 0.0600 \pm 0.0021$ M

EXAMPLE 2: In the formula below lower case letters represent positive constants (without error) and upper case letters positive values (with errors).

$$Z = \frac{aYX}{W} + \frac{bV+cU-e}{(T-S)} - fR + gQ + h$$

Provided $T_{Min} > S_{Max}$:

$$Z_{Max} = \frac{aY_{Max}X_{Max}}{W_{Min}} + \frac{bV_{Max}+cU_{Max}-e}{(T_{Min}-S_{Max})} - fR_{Min} + gQ_{Max} + h$$

$$Z_{Min} = \frac{aY_{Min}X_{Min}}{W_{Max}} + \frac{bV_{Min}+cU_{Min}-e}{(T_{Max}-S_{Min})} - fR_{Max} + gQ_{Min} + h$$

The ±Error in Z will be the larger of (Z_{Max}-Z) and (Z-Z_{Min}).

MANUFACTURER'S ERROR (For Class B Glassware)

Volume /ml	Volumetric Flask	Pipettes (Bulb)	Pipette (Graduated)	Burette	Measuring Cylinder
1		± 0.012	± 0.02		± 0.2
2		± 0.012	± 0.02		
5	± 0.04	± 0.020	± 0.05	± 0.02	± 0.2
10	± 0.08	± 0.040	± 0.10	± 0.05	± 0.2
20		± 0.060			
25	± 0.08	± 0.060	± 0.20	± 0.10	± 0.6
50	± 0.10	± 0.10		± 0.10	± 0.8
100	± 0.20	± 0.16		± 0.20	± 1.2
250	± 0.30				± 2.8
500	± 0.50				± 5.2
1000	± 0.80				±10

Note: For Class A Glassware: ½ the values.

REVIEW OF ERROR ESTIMATION & TREATMENT

DEFINITIONS

- **Errors** are categorized as three types: **Gross** (i.e. prevents usable data being obtained), **Systematic** (i.e. error has the same sign every time) and **Random** (i.e. random fluctuations due to the apparatus or user).
- **Precision** relates to Random errors (i.e. the degree of scatter of the obtained data) whilst **Accuracy** is a measure of how close the (average) reading is to the true value. *(Smaller Random Error = More Precise)*
- **Total Error** in Volumetric Glassware **= Manufacturers Error** (i.e. the inherent error in the manufacture of the item) **+ Fill Error** (i.e. the error associated with the user).

ESTIMATING ERRORS

Digital Displays (Stable): The error associated with each reading is half the **resolution (Res)** of the display, with the **resolution** being the smallest amount the display can change by. *Warning: 1-5ml autopipettors typically display to two decimal places, but often have set-point increments of 0.05 ml – This gives a resolution of 0.05ml and an error for each use of ±0.025 ml.*

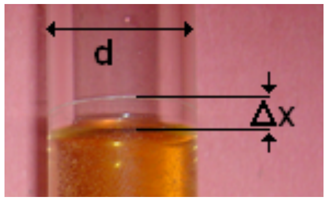
Digital Displays (Fluctuating): Estimate the range of the fluctuations. The value will be the mid-point of the range, with the reading error being half the range.

Volumetric Glassware (Graduated): Determine the **resolution**, i.e. the smallest variation in volume that you can differentiate with certainty.

Fill Error = ± ½Resolution

Warning: The resolution will not necessarily correspond exactly to the graduations on the glassware, since it is possible that you can differentiate volumes which are fractions of the way between graduations.

Volumetric Glassware (Single Fill Line): Determine the internal diameter (d) at the fill line and estimate the maximum distance (Δx) the liquid fill level could have been from the fill line.



Fill Error = ± $\frac{1}{4}\pi d^2\Delta x$

WHAT ERROR VALUES SHOULD BE INCLUDED IN LAB-BOOK

ITEM	ERROR INFO THAT SHOULD BE INCLUDED IN LAB-BOOK	ERROR INFO THAT "COULD" BE INCLUDED
Dig. Display (Stable)	Reading Error (i.e. $\pm\frac{1}{2}$**Res**) or **Res**	-
Dig. Display (Fluctuating)	Range of fluctuations	Reading Error (i.e. $\pm\frac{1}{2}$Range)
Vol. Glassware (Graduated)	**Manufacturers Error** and **Fill Error** (i.e. $\pm\frac{1}{2}$**Res**) or **Res**	**Total Error**
Vol. Glassware (Single Fill)	**Manufacturers Error**, **d** and **Δx**	**Fill Error**; **Total Error**

ERROR TREATMENT IN *EXCEL*

http://www.l4labs.soton.ac.uk/main/menu1.htm

The **Data Analysis** options in *Excel* that will most often be used are **Descriptive Statistics** and **Regression**.

- **Descriptive Statistics** is used to determine the mean of a set of repeat readings with their associated **±Error** (or **95% Confidence Level**).
- **Regression** is used to determine, for a set of x-y data, the gradient (**m**) and intercept of a trendline with their associated **Upper 95%** and **Lower 95%** **Confidence Limits**.

*IMPORTANT: **Regression** outputs **Confidence Limits**, whilst **Descriptive Statistics** outputs **Confidence Level** – Make sure you know the difference.*

Note: Most Mac users will need to refer to *http://www.l4labs.soton.ac.uk/tutorials/excel/eda.htm* for alternative procedures that will enable them to determine these **Confidence Levels** and **Confidence Limits** using *Excel for Mac*.

Column1	
Mean	21.32717
Standard Error	0.334818
Median	21.3115
Mode	#N/A
Standard Deviation	0.820134
Sample Variance	0.67262
Kurtosis	0.513465
Skewness	-0.56004
Range	2.34
Minimum	20
Maximum	22.34
Sum	127.963
Count	6
Confidence Level(95.0%)	0.860678

For the **Descriptive Statistics** output table shown opposite (performed on a set of repeat Temperature readings) the result would be quoted as: **T = 21.3 ± 0.9** °C

For the **Regression** output table shown below (for Temperature-Time data), the **Coefficient of X Variable 1** corresponds to the Gradient (**m**), with the **Upper 95%** and **Lower 95%** being the maximum and minimum values it can take.

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-10.57642	6.64236	-1.59227	0.18654	-29.01858	7.86573	-29.01858	7.86573
X Variable 1	3.60539	0.43942	8.20490	0.00120	2.38537	4.82542	2.38537	4.82542

To determine the error in the gradient, find the difference between the value and the limits and cite the largest difference as the error, e.g. for the above

Δ_{Upper} = |Upper 95% – m| = |4.82542 – 3.60539| = 1.22003

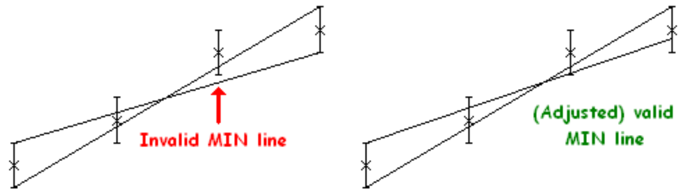
Δ_{Lower} = |m – Lower 95%| = |3.60539 – 2.38537| = 1.22002

Note: |…| means ignore any sign in calculation result.

m = 3.6 ± 1.2 °C/hr

*Note: The equivalent process can be used to determine the Intercept's **±Error**.*

PREFERABLY the error on the Gradient (**m**) and Intercept should be determined by **MIN/MAX LINES**: Having added appropriate error bars to the plot, add two new data series so trendlines (with equations) can be added that have the mininimum and maximum gradients (m_{Min} & m_{Max}). In the first instance these will connect the extreme error bars (i.e. top of the first point & bottom of the last point and vice versa), but they MUST then be MANUALLY ADJUSTED (if necessary) so they PASS BETWEEN the ERROR BARS of ALL the DATA being used. The larger of $|m_{Max}-m|$ and $|m-m_{Min}|$ will be the **±Error** of the gradient.



COMBINING ERRORS

To calculate a result when formula parameters have associated errors:

1. Calculate the **Result** ignoring the errors.
2. By adding and subtracting the errors to the parameters appropriately, recalculate to obtain the **Maximum** and **Minimum** possible values for the result.
3. Find the difference between the limits and the result (i.e. "**Maximum** - **Result**" and "**Result** - **Minimum**").
4. Cite the <u>**largest**</u> of the two differences as the **Results ±Error**.